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Corrections to Hawking-like Radiation for a Friedmann-Robertson-Walker Universe

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Abstract. Recently, a Hamilton-Jacobi method beyond semiclassical approximation in black hole physics was developed by *Banerjee* and *Majhi*[29]. In this paper, we generalize their analysis of black holes to the case of Friedmann-Robertson-Walker (FRW) universe. It is shown that all the higher order quantum corrections in the single particle action are proportional to the usual semiclassical contribution. The corrections to the Hawking-like temperature and entropy of apparent horizon for FRW universe are also obtained. In the corrected entropy, the area law involves logarithmic area correction together with the standard inverse power of area term.

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Inspired by black hole thermodynamics[1,2], it was realized that there is a profound connection between gravity and thermodynamics. In [3], Jacobson first showed that the Einstein equation can be derived from the proportionality of entropy to the horizon area, together with the Clausius relation $\delta Q = TdS$. Here δQ and T are the energy flux and Unruh temperature detected by an accelerated observer just inside the local Rindler causal horizons through spacetime point. Jacobson's derivation has also been applied to $f(R)$ theory [4] and scalar-tensor theory[5], where the non-equilibrium thermodynamics must be taken into account. For other viewpoint see [6].

With the spirit of Jacobson's derivation of Einstein field equation, one is able to derive Friedmann equations of a FRW universe with any spatial curvature by applying the Clausius relation to apparent horizon of the FRW universe. For FRW universe[7], after replacing the event horizon by the apparent horizon of FRW universe and assuming that the apparent horizon has an associated entropy S_{BH} and a temperature T_0

$$S_{\text{BH}} = \frac{A}{4\hbar}, \quad T_0 = \frac{\hbar}{2\pi\tilde{r}_A}, \tag{1}$$

one can turn the first law of thermodynamics, $dE = T_0 dS_{\text{BH}}$, to the Friedmann equations. Here $\hbar$, A, and $\tilde{r}_A$ are the Planck constant, area of the apparent horizon, and radius of the apparent horizon, respectively. Here it should be noted that the entropy S_{BH} and temperature T_0 are both the semiclassical results. The first law of thermodynamics not only holds in Einstein gravity, but also in Gauss-Bonnet gravity, Lovelock gravity, and various braneworld scenarios[8,9,10]. The fact that the first law of thermodynamics holds extensively in various spacetime and gravity theories suggests a deep connection between gravity and thermodynamics. (Some other viewpoints and further developments in this direction see [11,12,13,14,15,16] and references therein.)

Since we can view a FRW thermodynamical system, like as black holes[17], it is of great interest to ask that whether there is a Hawking-like temperature associated with the apparent horizon of FRW universe. Recently, the scalar particle and fermion's Hawking-like radiation from apparent horizon of FRW universe were investigated by using the semiclassical tunneling method[18,19]. The Hawking-like temperature $T_0 = \hbar/2\pi\tilde{r}_A$, which associated with the apparent horizon of FRW universe, was recovered.

The semiclassical tunneling process was initially proposed by Parikh and Wilczek[20]. In recent years, it has already attracted a lot of attention[21,22]. In Parikh and Wilczek's method, the imaginary part of the action is calculated with using the null geodesic equation. In addition to the null geodesic method, there is another method which was first developed by Padmanabhan et.al[23]. In this method, the Hawking radiation is derived by calculating the particles' classical action from the Hamilton-Jacobi equation. This method has been applied to more general and complicated spacetimes[24] and dynamics black holes[25], and also using this method, the tunneling of a Dirac particle through the event horizon was studied[26]. Later, the connection between the anomaly approach and tunneling formulism is also discussed[27]. Recently, the derivation of Hawking black body spectrum in the tunneling formulism is addressed[28] and this derivation fills the gap in the existing tunneling formulations. Both the

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null geodesic method and the Hamilton-Jacobi method are, however, confined to the semiclassical approximation only. The issue of higher order quantum corrections to the Hawking-like radiation from apparent horizon of FRW universe is generally not discussed.

Recently, an interesting improvement has already been made by *Banerjee* and *Majhi*[29]. They formulated the Hamilton-Jacobi method of tunneling beyond semiclassical approximation by considering all the terms in the expansion of the one particle action for a scalar particle, and obtained all the higher order quantum corrections to the semiclassical results. Some further applications of their method to other black holes, dynamics black holes and fermion tunneling also have been done[30]. However, examples given were mostly confined to black holes.

In this paper, we generalize Hamilton-Jacobi method of tunneling beyond semiclassical approximation of black holes to the case of FRW universe.[1] We also explicitly compute all the higher order quantum corrections to the Hawking-like temperature and the entropy of apparent horizon of FRW universe. Let us start with the standard FRW metric,

$$ds^2 = -dt^2 + a^2\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2\right), \tag{2}$$

where $d\Omega_2^2 = d\theta^2 + sin^2\theta d\varphi^2$ denotes the line element of an unit two-sphere S^2, a is the scale factor of our universe and k is the spatial curvature constant which can take values $k = +1$ (positive curvature), $k = 0$ (flat), and $k = -1$ (negative curvature). The metric (2) can be rewritten as

$$ds^2 = h_{ab}dx^a dx^b + \tilde{r}^2 d\Omega_2^2, \tag{3}$$

where $\tilde{r} = ar$ and $x^0 = t$, $x^1 = r$ and the two-dimensional metric h_{ab} =diag$(-1, a^2/(1-kr^2))$. In FRW universe, there is a dynamical apparent horizon, which is the marginally trapped surface with vanishing expansion and determined by the relation $h^{ab}\partial_a\tilde{r}\partial_b\tilde{r} = 0$. After a simple calculation one can obtain the radius of the apparent horizon

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + k/a^2}}, \tag{4}$$

where H is the Hubble parameter, $H \equiv \dot{a}/a$ (the dot represents derivative with respect to the cosmic time t). In the tunneling approach of reference [20] the Painlevé-Gulstrand coordinates are used for the Schwarzschild spacetime. Applying the change of radial coordinate, $\tilde{r} = ar$, along with the above definitions of H and $\tilde{r}_A$ to the metric in (2) one obtains the Painlevé-Gulstrand-like metric for the FRW spacetime

$$ds^2 = -\frac{1-\tilde{r}^2/\tilde{r}_A^2}{1-k\tilde{r}^2/a^2}dt^2 - \frac{2H\tilde{r}}{1-k\tilde{r}^2/a^2}dtd\tilde{r} + \frac{1}{1-k\tilde{r}/a^2}d\tilde{r}^2 + \tilde{r}^2 d\Omega_2^2. \tag{5}$$

These coordinates have been used in both null geodesic method and Hamilton-Jacobi method [18,19] to study the Hawking-like radiation from a FRW metric.

Consider the massless scalar field ϕ in the FRW universe, which obey the Klein-Gordon equation

$$\frac{-\hbar^2}{\sqrt{-g}}\partial_\mu(g^{\mu\nu}\sqrt{-g}\partial_\nu)\phi = 0. \tag{6}$$

Since FRW universe is spherical symmetric, we only interest in the $(t-\tilde{r})$ sector of the spacetime. By the standard ansatz for scalar wave function

$$\phi(\tilde{r}, t) = \exp\left[\frac{i}{\hbar}S(\tilde{r}, t)\right], \tag{7}$$

the Klein-Gordon equation (6) can be simplified to

$$\frac{\partial^2 S}{\partial t^2} + \left(\frac{i}{\hbar}\right)\left(\frac{\partial S}{\partial t}\right)^2 + \frac{H}{1-k\tilde{r}^2/a^2}\frac{\partial S}{\partial t} + \frac{\tilde{r}(H^2\tilde{r}_A^2 + 1 - k\tilde{r}^2/a^2)}{\tilde{r}_A^2(1-k\tilde{r}^2/a^2)}\frac{\partial S}{\partial \tilde{r}} - \left(\frac{i}{\hbar}\right)\left(1-\frac{\tilde{r}^2}{\tilde{r}_A^2}\right)\left(\frac{\partial S}{\partial \tilde{r}}\right)^2 + 2\frac{i}{\hbar}H\tilde{r}\frac{\partial S}{\partial \tilde{r}}\frac{\partial S}{\partial t} + 2H\tilde{r}\frac{\partial^2 S}{\partial t\partial\tilde{r}} - \left(1-\frac{\tilde{r}^2}{\tilde{r}_A^2}\right)\frac{\partial^2 S}{\partial \tilde{r}^2} = 0. \tag{8}$$

An expression of $S(\tilde{r}, t)$ in powers of $\hbar$ gives,

$$S(\tilde{r}, t) = S_0(\tilde{r}, t) + \sum_i \hbar^i S_i(\tilde{r}, t), \tag{9}$$

where $i = 1, 2, 3 \ldots$. In the semi-classical approach, we only consider the lowest term $S_0(\tilde{r}, t)$ and neglect the terms with $\hbar$ and greater. In this case, from (8) one can get the following equation,

$$(\partial_t S_0)^2 + 2H\tilde{r}\partial_t S_0 \partial_{\tilde{r}} S_0 - (1-\tilde{r}^2/\tilde{r}_A^2)(\partial_{\tilde{r}} S_0)^2 = 0, \tag{10}$$

and its solutions

$$\partial_t S_0 = (-H\tilde{r} \pm \sqrt{1-k\tilde{r}^2/a^2})\partial_{\tilde{r}} S_0. \tag{11}$$

The higher terms with $\hbar$ and greater are treated as quantum corrections to the semiclassical value S_0. Substituting (9) into (8) and using Eq.(11), after some calculations we find the following relations for different powers of $\hbar$,

$$\begin{aligned} \hbar^1 : &\qquad \partial_t S_1 = (-H\tilde{r} \pm \sqrt{1-k\tilde{r}^2/a^2})\partial_{\tilde{r}} S_1, \\ \hbar^2 : &\qquad \partial_t S_2 = (-H\tilde{r} \pm \sqrt{1-k\tilde{r}^2/a^2})\partial_{\tilde{r}} S_2, \\ &\qquad \vdots \end{aligned} \tag{12}$$

[1] We note that after submission of this manuscript, Ref.[37] appeared, which also treats the Hawking-like radiation in FRW universe by using the tunneling method beyond semiclassical approximation. Unlike [37] only considering the corrections to the Hawking-like temperature, we obtained the quantum corrections both to the semiclassical Hawking-like temperature and the entropy of apparent horizon.

and so on. The above set of equations have the same functional form. So their solutions are not independent and S_i are proportional to S_0. Then, we write the Eq.(9) by

$$S(\tilde{r},t)=(1+\sum_i \gamma_i \hbar^i)S_0(\tilde{r},t). \tag{13}$$

Here S_0 denotes the semiclassical contribution and the extra value $\sum_i \gamma_i \hbar^i S_0$ can be regarded as the quantum correction terms of the semiclassical analysis.

In order to find the solution of $S_0(\tilde{r},t)$ satisfying Eq.(11), one must analysis the symmetries of the metric (5). For the metric (5), since the metric coefficients are both radius and time dependent, there is no time translation Killing vector field in same with the case of static spacetime. However, following Kodama[31], for spherically symmetric dynamical spacetime whose metric like (5), there is a natural analogue, the Kodama vector

$$K=\sqrt{1-k\tilde{r}^2/a^2}\partial_t. \tag{14}$$

(For details of the definition of the Kodama vector and its significance, see [31,32].) The Kodama vector in dynamical spacetime is of the same significance with the Killing vector in static spacetime. It should be noted that the Kodama vector is timelike, null and spacelike as $\tilde{r}<\tilde{r}_A$, $\tilde{r}=\tilde{r}_A$ and $\tilde{r}>\tilde{r}_A$, respectively. Using the Kodama vector, one can define the energy ω and radial momentum $k_{\tilde{r}}$ measured by the Kodama observer

$$\omega=-K\partial_t S_0=-\sqrt{1-k\tilde{r}^2/a^2}\partial_t S_0, \quad k_r=\partial_{\tilde{r}}S_0. \tag{15}$$

Thus one can separate S_0 as

$$S_0=-\int\frac{\omega}{\sqrt{1-k\tilde{r}^2/a^2}}dt+\int k_{\tilde{r}}d\tilde{r}. \tag{16}$$

Substituting the above ansatz into Eq.(11) yields

$$k_{\tilde{r}}=\frac{-H\tilde{r}\pm\sqrt{1-k\tilde{r}^2/a^2}}{(1-\tilde{r}^2/\tilde{r}_A^2)\sqrt{1-k\tilde{r}^2/a^2}}\omega, \tag{17}$$

where the $+/-$ sign corresponds to the outgoing/intgoing solutions, respectively. Therefore solutions of action $S(\tilde{r},t)$ for the ingoing and outgoing particle under the background metric (5) are respectively,

$$\begin{aligned}S_{\mathtt{out}}(\tilde{r},t)=\Big[&-\int\frac{\omega}{\sqrt{1-k\tilde{r}^2/a^2}}dt+\\&\omega\int\frac{-H\tilde{r}+\sqrt{1-k\tilde{r}^2/a^2}}{(1-\tilde{r}^2/\tilde{r}_A^2)\sqrt{1-k\tilde{r}^2/a^2}}d\tilde{r}\Big]\\&\times(1+\sum_i\gamma_i\hbar^i),\end{aligned} \tag{18}$$

and

$$\begin{aligned}S_{\mathtt{in}}(\tilde{r},t)=\Big[&-\int\frac{\omega}{\sqrt{1-k\tilde{r}^2/a^2}}dt+\\&\omega\int\frac{-H\tilde{r}-\sqrt{1-k\tilde{r}^2/a^2}}{(1-\tilde{r}^2/\tilde{r}_A^2)\sqrt{1-k\tilde{r}^2/a^2}}d\tilde{r}\Big]\\&\times(1+\sum_i\gamma_i\hbar^i).\end{aligned} \tag{19}$$

Recently, a problem in tunneling approach has been discussed in which corresponds to a factor two ambiguity in the original Hawking temperature[33]. This ambiguity is resolved when we take into account a temporal contribution to the imaginary part of action[34]. In Schwarzschild black hole, for the tunneling of a particle across the event horizon the nature of the time coordinate t changes. This change indicates [34] that t coordinate has an imaginary part for the crossing of the horizon of the black hole and correspondingly there will be a temporal contribution to the imaginary part of action for the ingoing and outgoing particles. For FRW universe, the radiation is observed by the Kodama observer and the Kodama vector is timelike, null and spacelike for the regions outside, on and inside the apparent horizon, respectively. Because the energy of the particle is defined by the Kodama vector, the discrepancy of Kodama vector inside and outside the horizon will effect the temporal part of the action. This means that the temporal part integral in (18) and (19) also has an imaginary part. Therefore, outgoing and ingoing probabilities are given by,

$$\begin{aligned}P_{\mathtt{out}}&=|\phi_{\mathtt{out}}|^2=\left|\exp\left[\frac{i}{\hbar}S_{\mathtt{out}(\tilde{r},t)}\right]\right|^2\\&=\exp\Big[-\frac{2}{\hbar}(1+\sum_i\gamma_i\hbar^i)\Big(-\mathtt{Im}\int\frac{\omega}{\sqrt{1-kr^2}}dt\\&+\omega\mathtt{Im}\int\frac{-H\tilde{r}+\sqrt{1-kr^2}}{(1-\tilde{r}^2/\tilde{r}_A^2)\sqrt{1-kr^2}}d\tilde{r}\Big)\Big]\end{aligned} \tag{20}$$

and

$$\begin{aligned}P_{\mathtt{in}}&=|\phi_{\mathtt{in}}|^2=\left|\exp\left[\frac{i}{\hbar}S_{\mathtt{in}(\tilde{r},t)}\right]\right|^2\\&=\exp\Big[-\frac{2}{\hbar}(1+\sum_i\gamma_i\hbar^i)\Big(-\mathtt{Im}\int\frac{\omega}{\sqrt{1-kr^2}}dt\\&+\omega\mathtt{Im}\int\frac{-H\tilde{r}-\sqrt{1-kr^2}}{(1-\tilde{r}^2/\tilde{r}_A^2)\sqrt{1-kr^2}}d\tilde{r}\Big)\Big].\end{aligned} \tag{21}$$

The contribution of the temporal part of the action to the tunneling rate is canceled out when dividing the outgoing probability by the ingoing probability because the temporal part is completely the same for both the outgoing and ingoing solutions. It is no need to work out the result of the temporal part of the action.

In the WKB approximation, the tunneling probability is related to the imaginary part of the action as

$$\begin{aligned}\varGamma&=\frac{P_{\mathtt{in}}}{P_{\mathtt{out}}}\\&=\exp\left[\frac{4\omega}{\hbar}(1+\sum_i\gamma_i\hbar^i)\mathtt{Im}\int\frac{1}{(1-\tilde{r}^2/\tilde{r}_A^2)}d\tilde{r}\right].\end{aligned} \tag{22}$$

It is obvious that the integral function has a pole at the apparent horizon. Through a contour integral, the tunneling probability of ingoing particle now reads

$$\varGamma=\exp\left[-\frac{2}{\hbar}(1+\sum_i\gamma_i\hbar^i)\pi\omega\tilde{r}_A\right]. \tag{23}$$

Now using the principle of "detailed balance"[23],

$$\Gamma = \exp[-\omega/T] = \exp[-\omega/T], \tag{24}$$

the Hawking-like temperature associated with the apparent horizon can be determined as

$$T = \frac{\hbar}{2\pi\tilde{r}_A}\big(1+\sum_i \gamma_i \hbar^i\big)^{-1} = T_0\big(1+\sum_i \gamma_i \hbar^i\big)^{-1}, \tag{25}$$

where T_0 is the semiclassical Hawking-like temperature and other terms are corrections coming from the higher order quantum effects.

In the Hawking-like temperature expression (25), there are un-determined coefficients γ_i. Since S_0 has the dimension of $\hbar$, the coefficients γ_i should have the dimension of inverse of $\hbar^i$. In the units $G = c = k_B = 1$ the Planck constant $\hbar$ is of the order of square of the Planck length l_p. Therefore, the coefficients γ_i have the dimension of $\tilde{r}_A^{-2}$. We can write the action S as

$$S(\tilde{r}, t) = \left(1+\sum_i \frac{\alpha_i \hbar^i}{\tilde{r}_A^{2i}}\right) S_0(\tilde{r}, t), \tag{26}$$

where α_i are dimensionless parameters. Now the Hawking-like temperature (25) can be written as

$$T = T_0\left(1+\sum_i \frac{\alpha_i \hbar^i}{\tilde{r}_A^{2i}}\right)^{-1}. \tag{27}$$

Till now, we have obtained all the corrections to the semiclassical Hawking-like temperature T_0. It should be noted that the Kodama observer is inside the apparent horizon. This means that the Kodama observer does see a thermal spectrum with temperature $T = (1/2\pi\tilde{r}_A)(1+\sum_i \frac{\alpha_i \hbar^i}{\tilde{r}_A^{2i}})^{-1}$.

Hawking temperature is always related to surface gravity of horizon as $T = \kappa/2\pi$. In the semiclassical case, the surface gravity is $\kappa_0 = 1/\tilde{r}_A$. Hence, the modified form of surface gravity of the apparent horizon following from (27), is

$$\kappa = \kappa_0\left(1+\sum_i \frac{\alpha_i \hbar^i}{\tilde{r}_A^{2i}}\right)^{-1}. \tag{28}$$

Now let us turn to investigate the entropy of apparent horizon in the presence of higher order quantum corrections. The semiclassical Bekenstein-Hawking entropy of apparent horizon is given by

$$S_{\mathtt{BH}} = \frac{A}{4\hbar}. \tag{29}$$

The first law of thermodynamics holds on apparent horizon indicates $dS_{\mathtt{BH}} = dE/T_0$, where dE is the amount of energy crossing the apparent horizon in FRW universe. In constructing the first law of thermodynamics on apparent horizon, a key point is to calculate this energy dE in an infinitesimal time interval. In FRW universe, the total energy inside the apparent horizon is defined by a quasi-local mass: the Misner-Sharp mass $M = \tilde{r}_A/2$. By using the Misner-Sharp mass M, the energy flux passed through the apparent horizon is defined as

$$dE = (k^t\partial_t M + k^r\partial_r M)dt = d\tilde{r}_A, \tag{30}$$

where $k^{t,r} = (1, -Hr)$ is the (approximate) generator of the apparent horizon and satisfies $k^r\partial_r\tilde{r} + k^t\partial_t\tilde{r} = 0$. With the expression of modified Hawking-like temperature (27), the first law of thermodynamics on apparent horizon is

$$dS = \frac{dE}{T} = \frac{d\tilde{r}_A}{T_0}\left(1+\sum_i \frac{\alpha_i \hbar^i}{\tilde{r}_A^{2i}}\right). \tag{31}$$

Integrating the above equation yields the entropy of apparent horizon

$$\begin{aligned} S &= \int \frac{d\tilde{r}_A}{T_0}\left(1+\sum_{i=1} \frac{\alpha_i \hbar^i}{\tilde{r}_A^{2i}}\right) \\ &= \frac{A}{4\hbar} + \pi\alpha_1 \ln\frac{A}{4\hbar} + \sum_{i=2}\frac{\pi^i\alpha_i}{1-i}(\frac{A}{4\hbar})^{1-i} + \mathtt{const}. \end{aligned} \tag{32}$$

We can see that the first term is the usual semiclassical area law (29), and the other terms are the quantum corrections. For the correction term, it contain two parts: the logarithmic term and the inverse area terms. We note that the logarithmic correction term has been also obtained by other approaches[11,35,36], and in some literatures[36], the coefficient of the logarithmic correction term is controversial. In our result, the coefficient of the logarithmic correction term is determined by the dimensionless constant α_1.

In conclusion, we generalize Hamilton-Jacobi method of tunneling beyond semiclassical approximation of black holes to the case of FRW universe. We have considered all orders in the single particle action for particle tunneling through the apparent horizon of the FRW universe. It is shown that higher order correction terms of the action are proportional to the semiclassical contribution. By applying the dimensional argument and principle of "detailed balance", higher order corrections to the Hawking-like temperature and entropy of apparent horizon are obtained. For the corrected entropy (32), it contains three parts: the usual Bekenstein-Hawking entropy, the logarithmic term and the inverse area term. We find that the coefficient of the logarithmic correction term is determined by the dimensionless constant α_1.

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